

# Logos: Logic in Computer Science

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# Plan

## 1. Algorithmic Game theory

- Simultaneous games
  - 0-sum games
  - Cooperative games, Nash equilibria
  - Auctions
- Sequential games
  - Markov Decision Processes: MDPs
  - 2 players

## 2. LLM's

- Tokens
- Language model
- RLHF

# 1. Algorithmic game theory

Cooperative games

<http://mdr.free.fr/td/PL14.ppt>

<http://dup2.free.fr/index.php?n=Main.BD10>

- 0-sum games: linear programming, fictitious player
- General games: Nash equilibria, Hemke-Lowson
- Auctions: Vickrey auctions
- Adwords

# Algorithmic game theory: MDPs

MDPs: Markov Decision processes, Sequential Games

<http://dup2.free.fr/index.php?n=Main.BD10bis>

- Games against nature, 1 +  $\frac{1}{2}$  players
- 2 players: chess, go
- N-players (robots, autonomous vehicles)

# Reinforcement Learning

## 1. Search for optimal strategies in MDP: Q-learning

- Value iteration  $V^*(s) = \max_{\pi} V^{\pi}(s)$  Bellman's equation
- Policy iteration  $\pi^*(s) = \operatorname{argmax}_a \sum_{s'} P_{s,a}(s') V^*(s')$
- Q-learning:  $V(s) = \max_a Q(s,a)$   
 $a = \operatorname{arg max}_a Q(s,a)$

## 2. Learning with experts:

<http://dup2.free.fr/index.php?n=Main.BD10a>

## 2. LLMs

1. Tokens and syntax

1. Language model

1. RLHF: Reinforcement Learning with Human Feedback

<https://stanford-cs324.github.io/winter2022/>

<https://stanford-cs324.github.io/winter2023/>

# 2.1 Tokens

Morphologie: *re-construc-tion*

Entités Nommées: *First National Bank of Chicago*

Formes: *il y a , tout à coup*

Mots sémantiques: *machine à laver*

Lemmatisation: *racine, forme canonique*

BPE: Byte Pair Encoding

# Tokens: BPE

Phrase d'origine : "abracadabra »

Initialisation :

"a", "b", "r", "a", "c", "a", "d", "a", "b", "r", "a"

Étape 1 : Fusion des paires les plus fréquentes :

"ab", "r", "a", "c", "a", "d", "a", "b", "r", "a"

Étape 2 : Nouvelle fusion :

"abr", "a", "c", "a", "d", "a", "b", "r", "a"

Étape 3 : Nouvelle fusion :

"abra", "c", "a", "d", "a", "b", "r", "a"

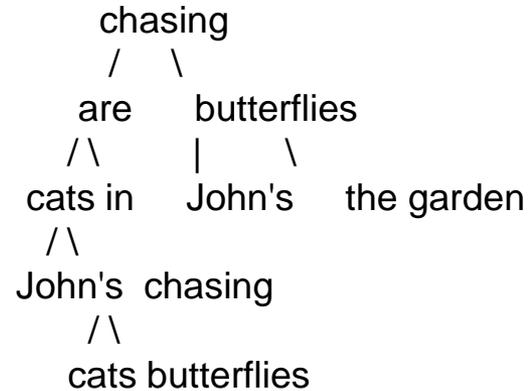
Étape 4 : Dernière fusion :

"abrac", "a", "d", "a", "b", "r", "a"

# Structure des tokens

"John's cats are chasing butterflies in the garden."

Arbre syntaxique :



Lemmatisation :

"John's" devient "John" (suppression de la possession et de l'apostrophe).

"cats" reste "cat" (réduction à la forme de base).

"are" reste "are" (verbe auxiliaire).

"chasing" devient "chase" (réduction à la forme de base).

"butterflies" devient "butterfly" (réduction à la forme de base).

"in" reste "in" (préposition).

"the" reste "the" (article défini).

"garden" reste "garden" (nom).

## 2.2 Language Model

Représentation de la distribution

$U_{stat}_k(w_1 \dots w_k)$  et  $Next_k(w_1 \dots w_k)$

Forme compressée, grâce à 3 Embeddings

(Word2vec) Query, Key, Value et à **l'attention**:

John's cats are chasing ?

$v_1$        $v_2$        $v_3$        $v_4$

Matrice:  $M(i,j) = v_i * v_j$

# Language Model

Attention Scores:

Butterflies  $\text{Sum}_i Q_{\text{Butterflies}} * K_i = 5$

Mice:  $\text{Sum}_i Q_{\text{Mice}} * K_i = 3$

Softmax (3,5):  $e^5 / e^3 + e^5$  ,  $e^3 / e^3 + e^5$

John's cats are chasing ?

$V_1$        $V_2$        $V_3$        $V_4$

### 3. RLHF



"Un coucher de soleil éclatant sur une plage déserte."

Poids : 0.2

"Un paysage urbain avec des gratte-ciels illuminés la nuit. Un paysage urbain avec des gratte-ciels illuminés la nuit."

Poids : 0.8

# Supervised fine-tuning 2021,22,23...

Other methods:

1. Best-of-n
2. Expert iteration
3. Conditional tokens
4. Contrastive-based methods

# Market for CHatBots

1. ChatGPT
2. Bard
3. Llama
4. Huggingface

Less known:

You: [www.you.com](http://www.you.com)

New: Mistral

Citer ses sources  
Interface BD

# Conclusion

1. Algorithmic Game theory
  - Cooperative games
  - Mechanisms and Equilibria
2. LLM's
  - Tokens
  - Language Model
  - RLHF

# Testers and Correctors

Distance: Edit, Edit with moves,....

$\epsilon$ -close

1. Words: regular expressions,  $U = (\{1, 2, \dots, n\}, <, A, B)$

$w = ababacababacacab$

$r = (ab)^*(ac)^*$

Corrector:  $w_1 = ababababacac$

$w_2 = ababa**b**ababacacac$

$w_3 = ababa**b**ababacac$

$w_4 = ababababacacac$

$Q(x)$ :  $A(x)$  and  $B(x+1)$  then  $CQA(A) = \{1, 3, 5, 7\}$